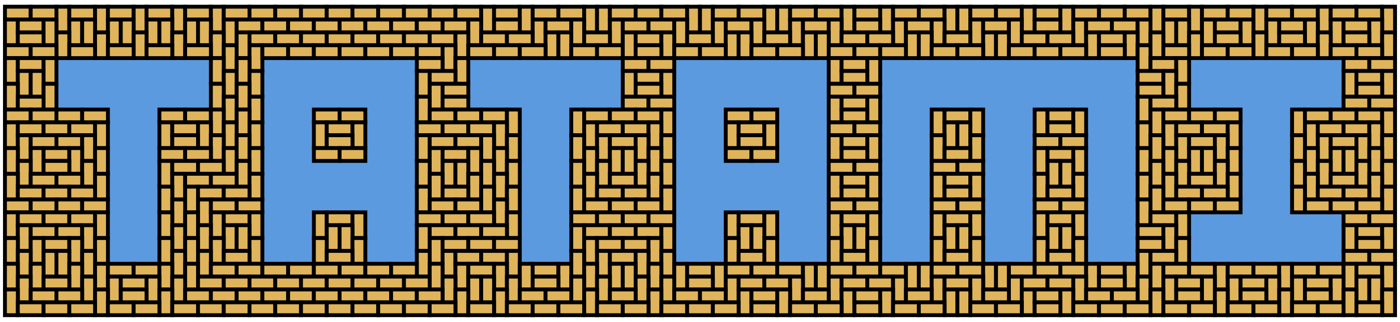


No four tiles meet!

The Brilliant Club

How to Make a Mathematical Discovery

Key Stage 4 -- Spring 2015

Tutor Name

Pupil Name ______________________

The Scholars Programme – Spring 2015 – Pupil Feedback Report

Grade	Marks	What this means
1st	70+	Performing to an excellent standard above current key stage
2:1	60-69	Performing to a good standard above current key stage
2:2	50-59	Performing to an excellent standard at current key stage
3rd	40-49	Performing to a good standard at current key stage
Workings towards a pass	0-39	Workings towards a pass
Did not submit	DNS	No assignment received by The Brilliant Club

Lateness	
Any lateness	10 marks deducted
Plagiarism	
Some plagiarism	10 marks deducted
Moderate plagiarism	20 marks deducted
Extreme plagiarism	Automatic fail

Name of PhD Tutor			
Title of Assignment			
Name of Pupil			
Name of School			
ORIGINAL MARK / 100		**FINAL MARK / 100**	
DEDUCTED MARKS		**FINAL GRADE**	

If marks have been deducted (e.g. late submission, plagiarism) the PhD tutor should give an explanation in this section:

Learning Feedback Comment 1 - ***Enter Key Learning Priority Here***	
What you did in relation to this Key Learning Priority *Enter feedback here*	**How you could improve in the future** *Enter feedback here*
Learning Feedback Comment 2 – ***Enter Key Learning Priority Here***	
What you did in relation to this Key Learning Priority *Enter feedback here*	**How you could improve in the future** *Enter feedback here*
Learning Feedback Comment 3 – ***Enter Key Learning Priority Here***	
What you did in relation to this Key Learning Priority *Enter feedback here*	**How you could improve in the future** *Enter feedback here*
Resilience Comment	
How you showed learning resilience during the course *Enter feedback here*	**How you could build learning resilience in the future** *Enter feedback here*

Table of Contents

Course Rationale

Paddington Bear is preparing his suitcase for a picnic lunch, and even with its secret compartment he can't quite get all of the delicious foods into it that he wants to! In order to maximise deliciousness, he needs to make careful choices about what to take with him and what to leave behind. Combinatorics can be used to solve Paddington's problem and many other more sophisticated ones, like how to build supercomputers, schedule airplane flights, solve Sudoku puzzles. One area of combinatorics pertains to arranging individual objects such that the arrangement as a whole exhibits a mathematically harmonious structure; such as the way certain molecules arrange themselves to make crystals. We will explore this area through the lens of my PhD research on tatami mat arrangements.

Japanese tatami mats are traditional floor furnishings that measure approximately 1m x 2m. In an auspicious arrangement they must satisfy **the tatami restriction: no four of their corners may meet**. We can pose several combinatorial problems about arranging tatami mats: How many ways are there to cover my 8m x 8m room floor with tatami mats? Hint: very few. What if I do not require the tatami restriction? Hint: very many.

In order to tackle this combinatorial problem, we will first discover the structure of tatami mat arrangements; the tatami restriction, it turns out, has a lot to say about how the mats can be arranged! We will learn about the process behind the mathematical discovery of the tatami structure by gaining an intuition for the tatami restriction through mathematical games, making inferences from data, deriving a theorem and proof of the tatami structure, and enumerating tatami mat arrangements by counting strings of 0s and 1s.

Mark Scheme

Key Skill: Application
1st
Pupil recognises combinatorial structures and describes them in the abstract.
Pupil recognises how and where combinatorial models are useful, and can construct their own examples to support this.

2.2
Pupil recognises combinatorial structures, and demonstrates limited abilities to abstract useful combinatorial structure.
Pupil recognises that combinatorial models are useful, and demonstrates understanding of most of the examples given in the course handbook.

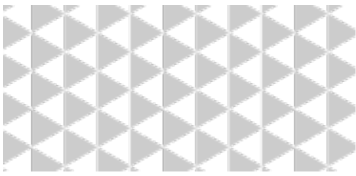
Key Skill: Communication
1st:
Complex ideas and arguments are clearly presented.
Key terms are used correctly.
Sophisticated mathematical arguments are presented in writing such that the combination of symbolic expressions, references to figures, and English has a natural, grammatically correct flow.
2.2
Complex ideas and arguments are attempted.
Attempts are made at using key terms, but sometimes these are misunderstood and used incorrectly.
Sophisticated mathematical arguments are presented in writing, where little context is given to symbolic expressions, or the written arguments are overly verbose.

Key Skill: Logical Reasoning
1st
Complex, multi-case logical arguments are followed, and the pupil confidently answers knowledge-testing questions.
Pupil presents logical arguments about key course concepts correctly and confidently.
Pupil spots false arguments and gaps in proofs, and can offer corrected arguments.
2.2
Complex, multi-case logical arguments are followed with limited understanding of the most challenging concepts, or how the whole of the logical argument supports the conclusion.
Pupil presents logical arguments with limited use of key course concepts, and which may not completely support the conclusion.
Pupil sometimes spots false arguments and gaps in proofs.

Key Skill: Research Creativity
1st
Pupil shows a creativity and resourcefulness that fosters mathematical discovery.
Pupil makes guided mathematical discoveries by creatively applying key concepts, and expanding on the course material.
2.2
Pupil shows limited creativity and resourcefulness.
Pupil makes simple, guided mathematical discoveries by directly applying key concepts.

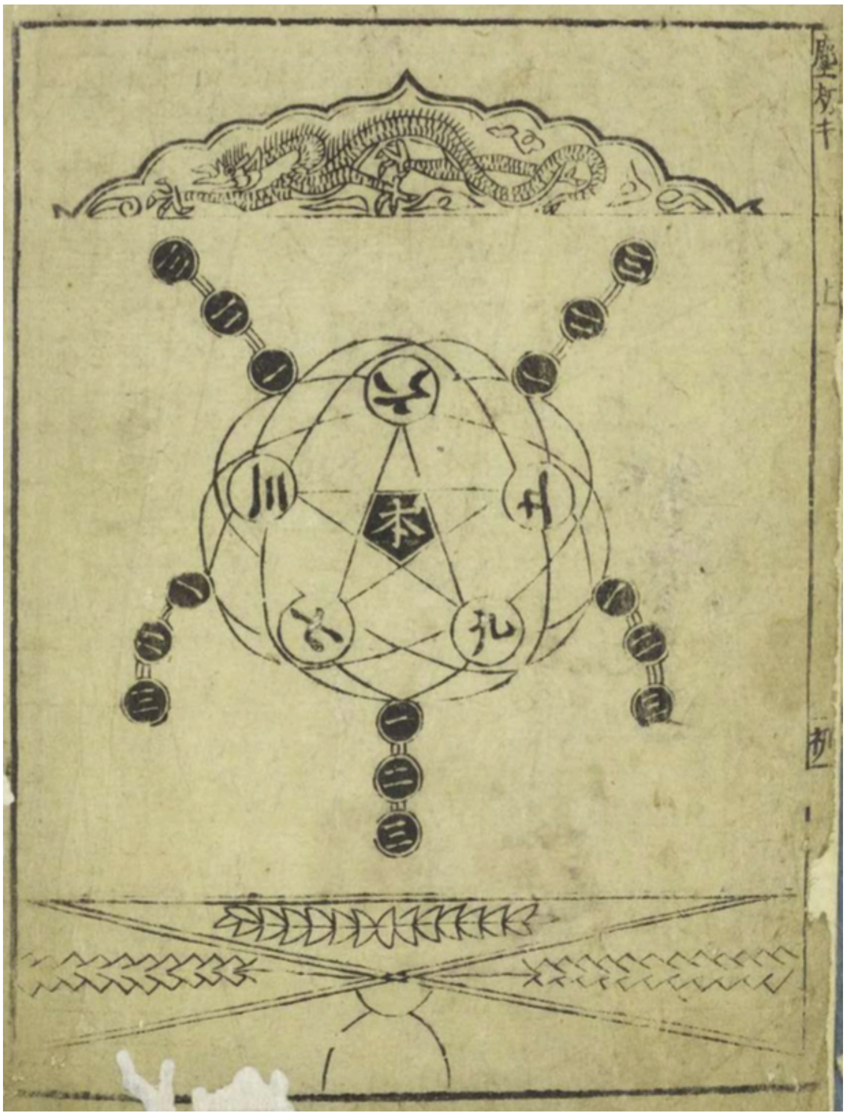
Glossary of Keywords

N.b., some of these definitions are credited to Google, Wikipedia, and Dictionary.com.

bidimer	A certain configuration of tiles, to be revealed in Tutorials 2-3.
binary string	A string of 1s and 0s, e.g., 1001001111, and 01.
box	The isometric grid's equivalent of a rectangle. See Tutorial 5 Activity 3.
combinatorics	The branch of mathematics that covers the material, and much, much more, in this programme. See also, discrete mathematics.
counting	A branch of research mathematics wherein arrangements of individual objects are counted, either exactly or approximately. This sometimes has physical applications, where the number of arrangements possible in the (discrete) mathematical model gives useful information about what is possible in the physical world.
covering	We say that tiles *cover* a region when the tiles do not overlap, the whole region is covered, and nothing but the region is covered.
diagonal flip	An alternate way of interpreting certain parts of the T-diagram tatami structure. See Tutorial 4 Activity 1.
discrete	Individually separate and distinct; "speech sounds are produced as a continuous sound signal rather than discrete units".
discrete mathematics	**T**he study of **mathematical** structures that are fundamentally **discrete** rather than continuous.
domino	A 1x2 rectangular tile (see **tile**).
enumeration	See **counting**.
global structure	The arrangement of the parts or elements of something complex.
grid	A rectangle divided up into unit squares, like a chessboard, or a sheet of grid paper.
isometric grid	A type of grid that can be drawn by repeating equilateral triangles. We use a checkered version for our own convenience. 
local rule	The relations between the parts of elements of something complex.
loner	A certain configuration of tiles, to be revealed in Tutorials 2-3.
lozenge	A tile that fits on the isometric grid, covering two adjacent triangles.
monomino	A 1 x 1 square tile. (see **tile**).
OEIS.org	The Online Encyclopaedia of Integer Sequences (http://oeis.org/) is a website that catalogues sequences of integers, like the Fibonacci sequence: 1,1,2,3,5,8,13,21, … .
Oku!	Players take turns putting tiles on a grid so that their opponent cannot make a move. Each tile that is placed must not violate the tatami restriction.

proof	Evidence or argument establishing a fact or the truth of a statement.
Q.E.D.	stands for the latin phrase ***quod erat demonstrandum***, meaning "which had to be proven". This is often used at the end of a mathematical proof.
ray	See Tutorial 2 Activity 3.
rhombus (tile)	See **lozenge**.
sequence	A sequence is a finite or infinite list of numbers, like the (infinite) Fibonacci sequence: 1,1,2,3,5,8,13,21, …, or the positive integers, 1,2,3,4, …. See **OEIS.org**.
T-diagram	An efficient diagram for expressing a monomino-domino tatami covering instead of showing all tiles. The T-diagram only shows the boundaries between vertical and horizontal dominoes.
tatami mat	A floor mat of Japanese origin, measuring approximately 1 meter by 2 metres (1 x 2 m).
tatami restriction	The tatami restriction for monomino-domino coverings of a rectangular grid says that no four tiles may meet at any point.
theorem	A general proposition not self-evident but proved by a chain of reasoning; a truth established by means of accepted truths.
tile	An 2-dimensional object of a fixed shape, which may represent a Tatami mat, for example, and may be placed with other tiles to cover an area.
Tomoku!	A pencil-and-paper game about finding tatami coverings.
triangle (tile)	A tile that fits on the isometric grid, covering one equilateral triangle.
triangle, grid-	A grid-triangle is a "grid square" of the isometric grid.
vee	A certain configuration of tiles, to be revealed in Tutorials 2-3.
vortex	A certain configuration of tiles, to be revealed in Tutorials 2-3.

Tutorial 1 – What is tatami?



A page from *Jinkōki,* the 17th century textbook on the soroban, a Japanese version of the abacus.

What is the Purpose of Tutorial 1?

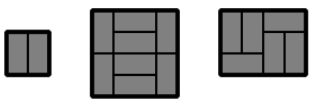
- To introduce monomino-domino coverings of rectangular grids.
- To see an application of monomino-domino coverings of rectangular grids.
- To introduce the tatami restriction.

Supplement

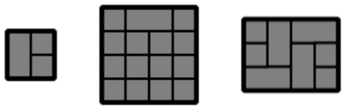
WARNING: These tutorials contain maths like you have never seen. There is easier stuff mixed with more complicated stuff, for those wanting more, but you are not expected to understand it all in 6 short tutorials!

A domino is a rectangular tile that measures 1x2 units. A domino can be placed on a grid so that it covers exactly two adjacent grid squares.

When we place enough dominoes in the grid to cover it completely (with no overlapping dominoes), we call this a *domino-covering* of the grid. Here are three examples.



A monomino is a square tile that covers exactly 1 grid square. We can place monominoes in the grid as well, to form monomino-domino coverings of the grid. Here are three examples.

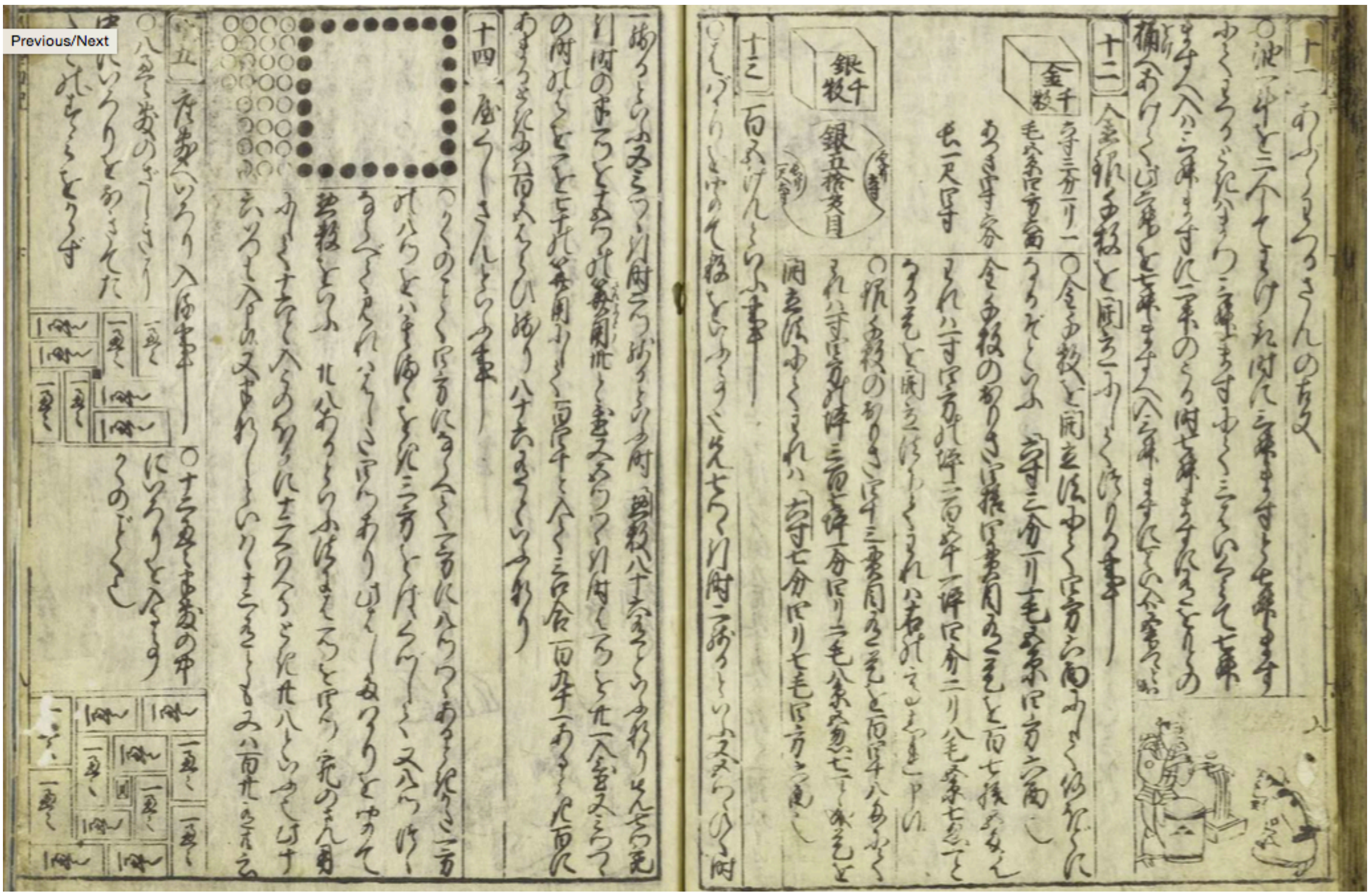


These coverings of grids describe certain situations in physics, e.g., where molecules are absorbed into discrete locations on a surface. Dominoes represent the molecules, and the surface has grid-like properties because of the forces that act at the atomic and molecular levels. This is an example of a **discrete structure**.

Tatami mats are a common floor furnishing, originating in aristocratic Japan, made with a rice straw core and a soft, woven rush straw exterior; they typically measure about 1x2m. In the 17th century, a simple rule was established for certain arrangements of tatami mats, which we call **the tatami restriction: no four mats may meet**.

Notice that "no four mats may meet" is actually a restriction at grid intersections, since the only way for four mats to meet is at their corners, at grid line intersections.

Historical anecdote: A Japanese mathematician named Mitsuyoshi Yoshida noticed the mathematical significance of tatami coverings, and published a drawing of one in his 1641 book on mathematics. The book, known as *Jinkōki,* describes how to use a Japanese abacus, as well as make many practical agricultural calculations like crop yields. Here is a page with a tatami mat arrangement shown on the left.



Activities

1. **Monomino-domino coverings of the grid:** Rectangular grids will be laid out at stations, and we will cover them with monominoes and dominoes by placing tiles in the grid at each station.
   Activity questions:
   a. What is a monomino? What is a domino?
   b. What is a domino covering of the grid? What is a monomino-domino covering of the grid?
   c. Are there domino coverings that differ from the ones we found in the activity?
   d. How many domino coverings are there? What is your best guess?
   e. Is the number of arrangements allowing both monominoes and dominoes be greater or fewer than those with only dominoes.
   f. Is it possible to "get stuck" trying to cover the grid with dominoes? With monominoes and dominoes?
2. **The tatami restriction:** We will repeat activity 1, but this time we will use the **tatami** restriction; **no four tiles meet**.
   Activity questions:
   a. Was it easier or more difficult to cover the grid using the tatami restriction?
   b. Describe how one might "get stuck" trying to make a tatami covering.
   c. Are there more or fewer tatami-restricted coverings than normal coverings using the same types of tiles? Guess at how many tatami coverings there might be of the type we tried to find in this activity.

Tutorial 1 – Baseline Test

The homework assignment for the first tutorial is a baseline test to see your initial level of attainment in this subject area. The assignment will test for some or all of the subject specific skills that are required later in the final assignment. However, it is shorter than the final assignment and is will be an introduction to the subject as well as a challenge!

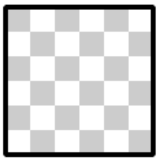
Do not worry too much about doing 'well' or 'badly' on the baseline test, it takes into account the fact that you may not be familiar with the subject area. It is designed to help you and your PhD tutor identify where you are at the start of the programme and to help you measure your progress along the way.

1. **True False and Difficulty level.**
Circle True or False for each question, and rate the difficulty of the question. You can draw in these grds or use the grid paper in the Appendix to do some rough work, if necessary.

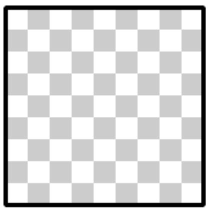
Can the grids below be tatami covered by dominoes (only)?

a. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)

b. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



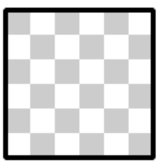
c. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



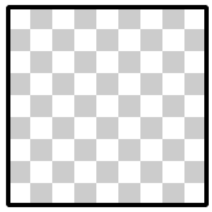
Can the grids below be tatami covered by dominoes and *exactly one* monomino?

d. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)

e. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)

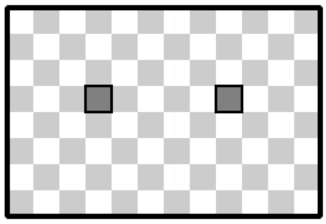


f. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)

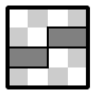


Can the partial *tatami* coverings below be completed with monominoes and/or dominoes?

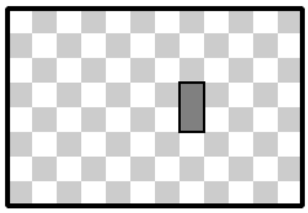
g. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



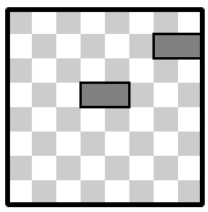
h. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



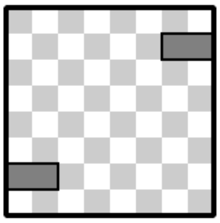
i. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



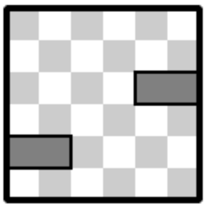
j. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



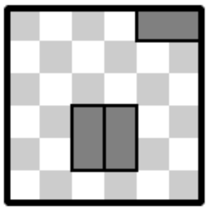
k. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



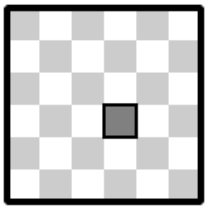
l. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



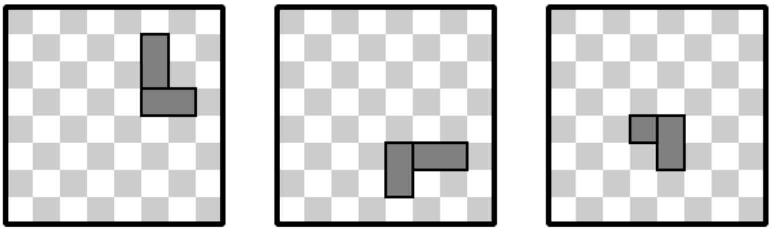
m. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



n. Circle one: True False Circle one: (easy) 1 2 3 4 5 (difficult)



2. For each of these partial tatami coverings, draw the tiles whose placements are forced by the tatami restriction.



3. Did you discover anything about tatami coverings? If you did, describe your discovery in a few sentences (and figures, if needed).

Tutorial 2 – Combinatorial structure

Tomoku!

What is the Purpose of Tutorial 2?

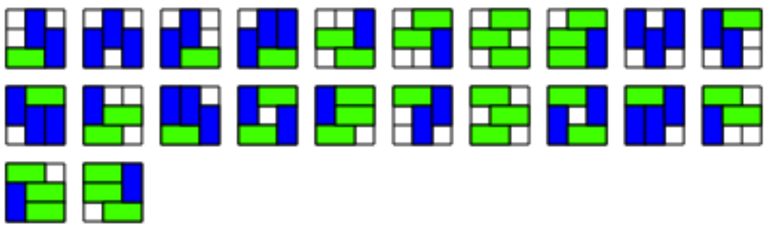
- To learn one of the processes of mathematical discovery; identifying patterns and describing them using mathematics.
- To learn the structure of tatami coverings and become familiar with it.
- To construct a complex logical argument; theorem and proof.

What is a theorem and proof?

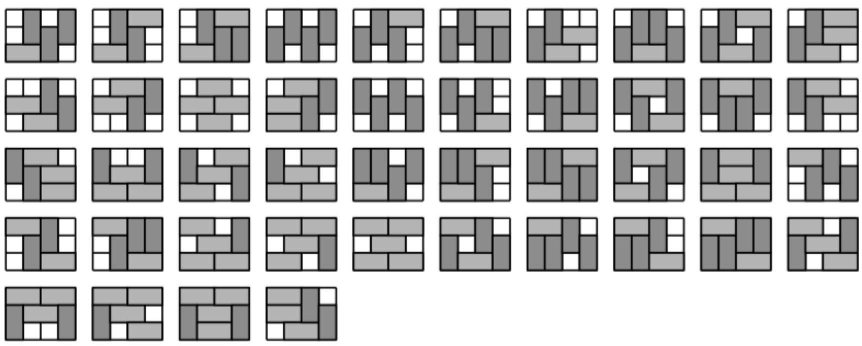
Many mathematical discoveries are given as a short standalone statement, called a *theorem* followed a proof, using deductive reasoning, of why the theorem is irrefutably true. The proof is expected to build up to the conclusion by combining simpler claims that the intended audience will understand and be able to accept as true.
Question: How is a theorem different from a theory?

Activities

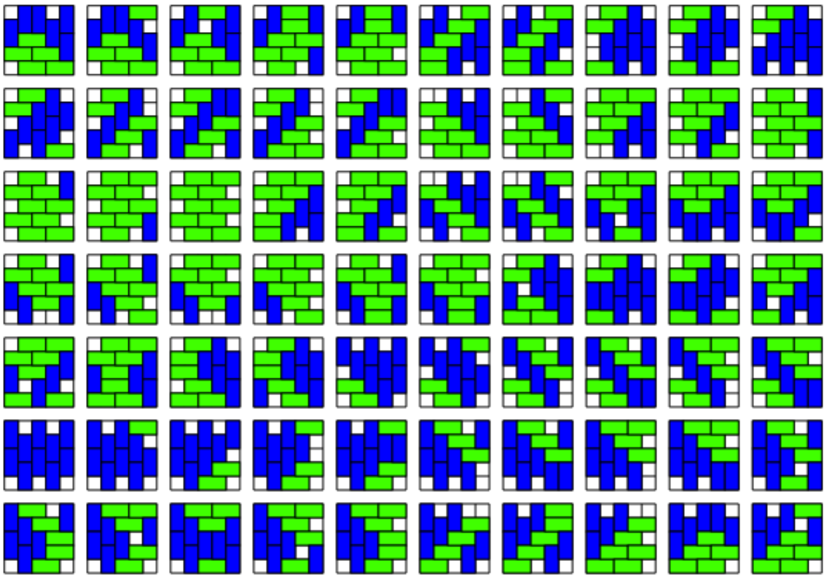
1. **Tomoku:**
   The object of the Tomoku puzzle is to tatami cover the grid with the tiles given for each row and column. We'll do this together in the tutorial, but full instructions and extra puzzles are included in the Appendix.
2. Discovering mathematics: Mathematicians often discover new maths by examining data. **Look at as many drawings of tatami coverings as possible** and try to identify similarities between them, tile configurations that determine other parts of the tatami covering, etc. **Discuss your observations in groups of 3**. Some tatami coverings are given below to get your thoughts started.



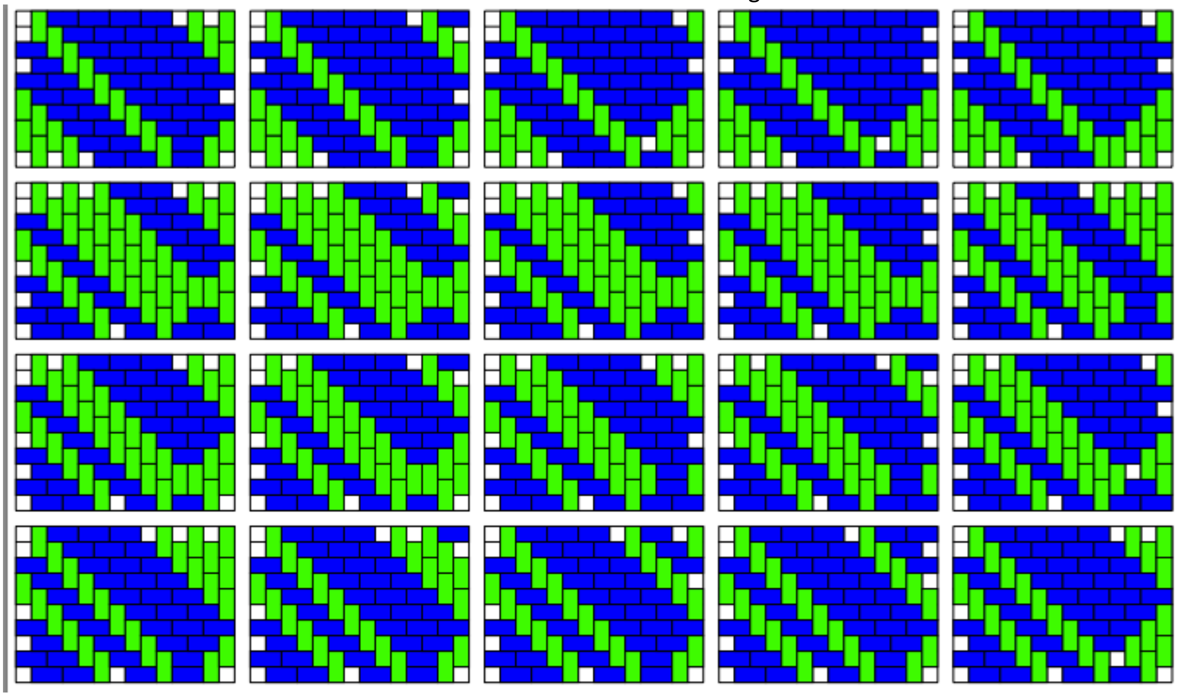
Every possible 3x3 (monomino-domino) tatami covering



Every possible 3x4 (monomino-domino) tatami covering

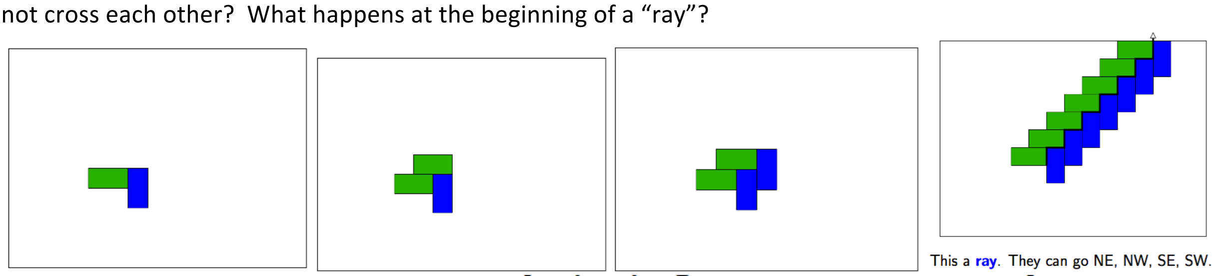


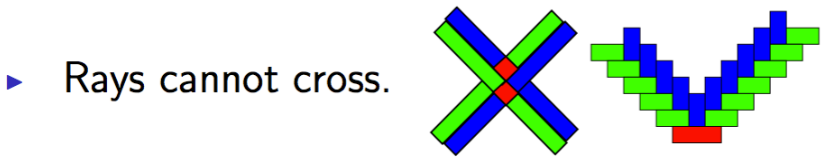
Some of the 5x5 tatami coverings.



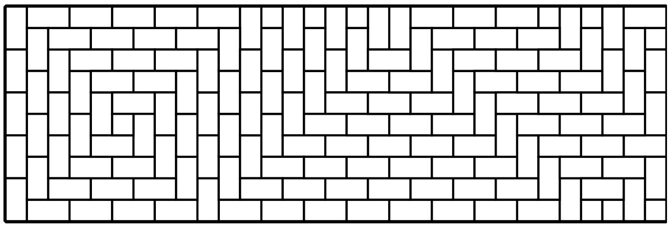
Some of the 10x14 coverings.

3. **Rays**: This configuration propagates by forcing the placement of other tiles until a boundary is reached. Why can rays not cross each other? What happens at the beginning of a “ray”?





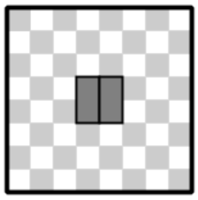
4. Identify the sources (“beginnings of rays”) in a tatami covering.



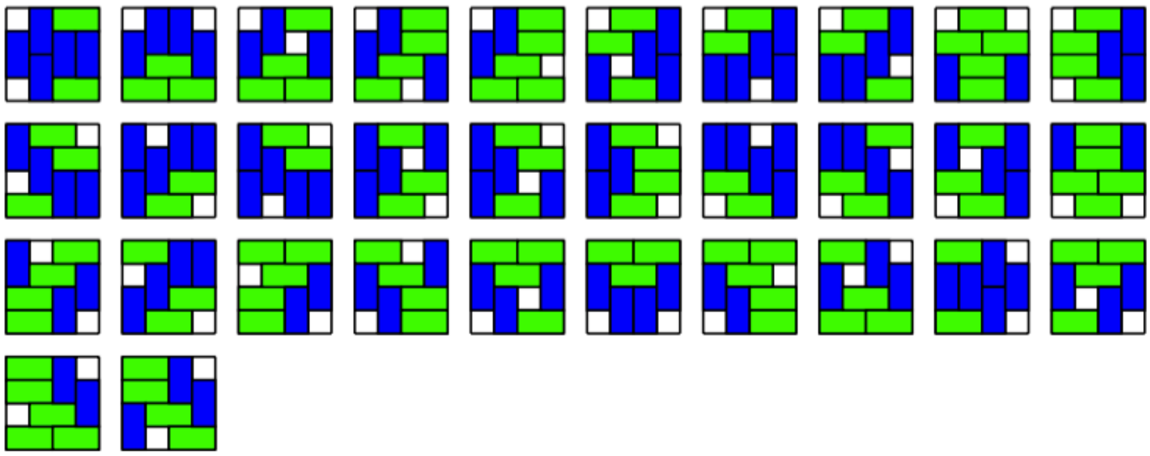
5. We will state the global structure of tatami coverings in a **theorem** and **prove** it together.

Homework

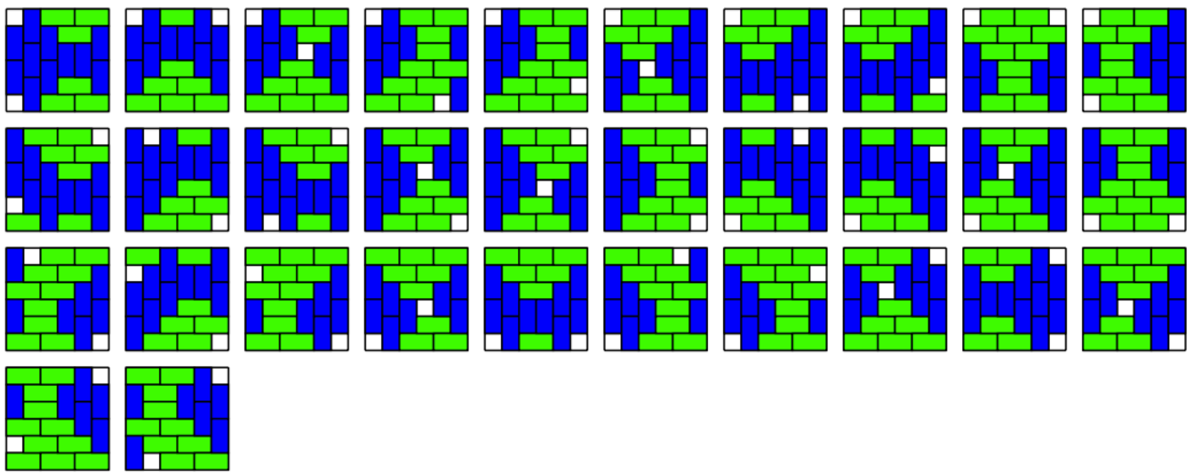
1. **Tomoku:** Complete two Tomoku puzzles of your choice, including one that is at least 5 rows tall. Find them in the Appendix.
2. Use the structure to argue that this covering can only be completed in one way).



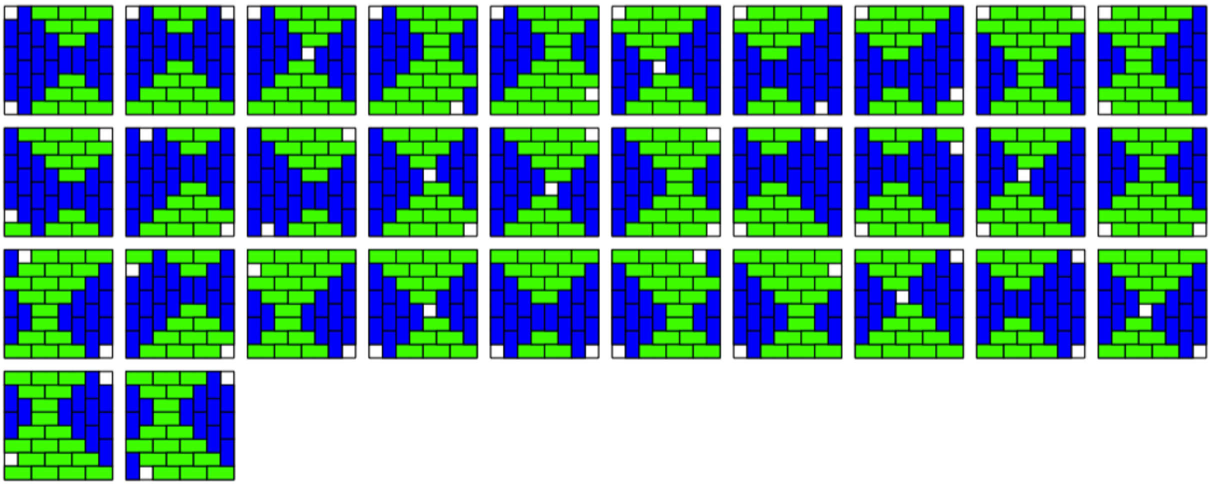
3. Construct a covering of any grid size using exactly one of each type of “feature” (see handout).
4. Study these different sized coverings. Given what you know of the tatami structure, and that the first image shows **all 32** of the 4x4 tatami coverings with exactly 2 monominoes, what can you say about the larger coverings? Are all of them shown here? Notice only even numbers are shown. How many 9x9 tatami coverings with exactly 2 monominoes are there?



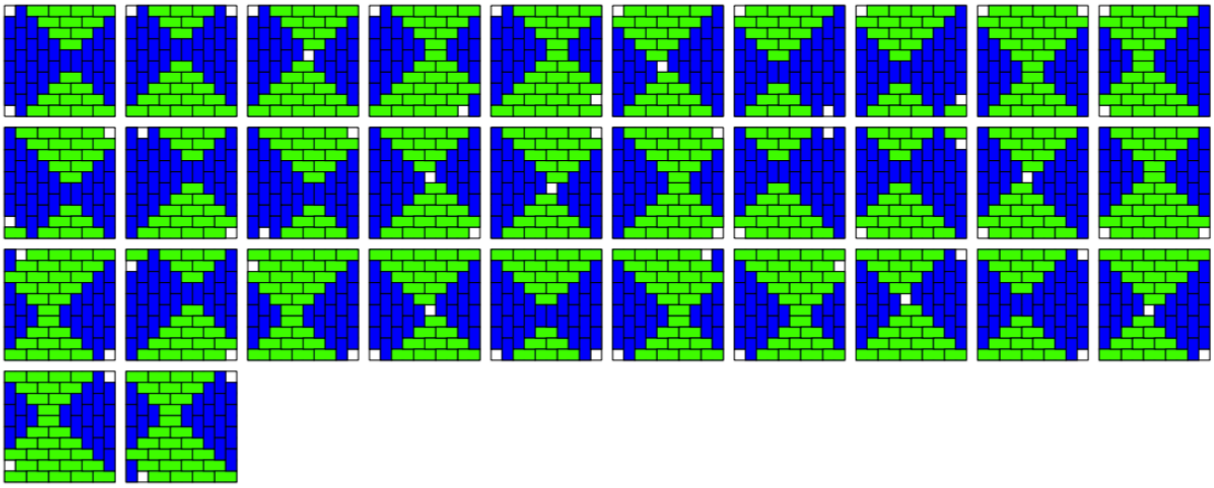
All 32 of the 4x4 tatami coverings with exactly 1 monomino.



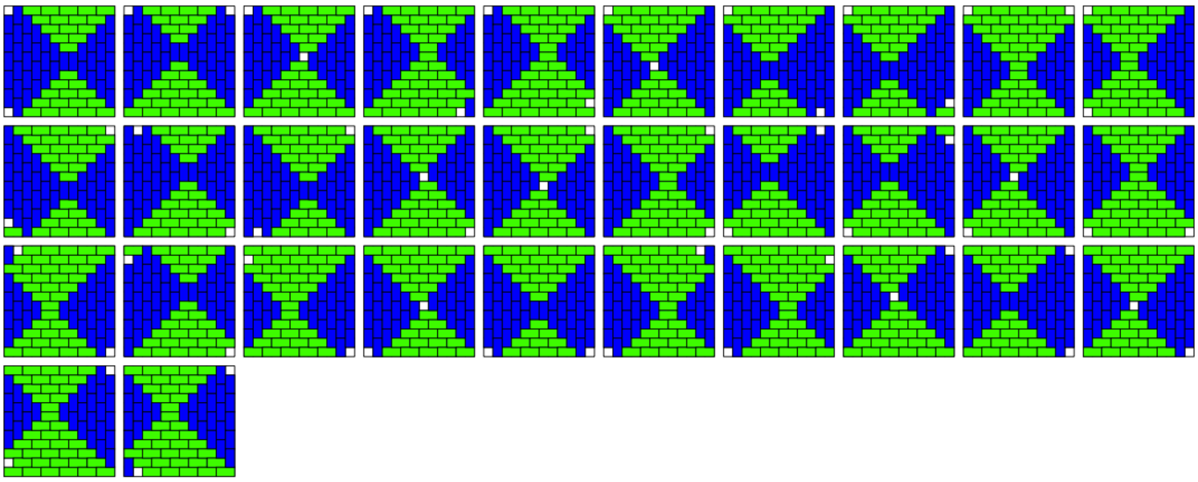
6x6 tatami coverings with exactly 2 monominoes. Is this all of them?



8x8 tatami coverings with exactly 2 monominoes. Is this all of them?

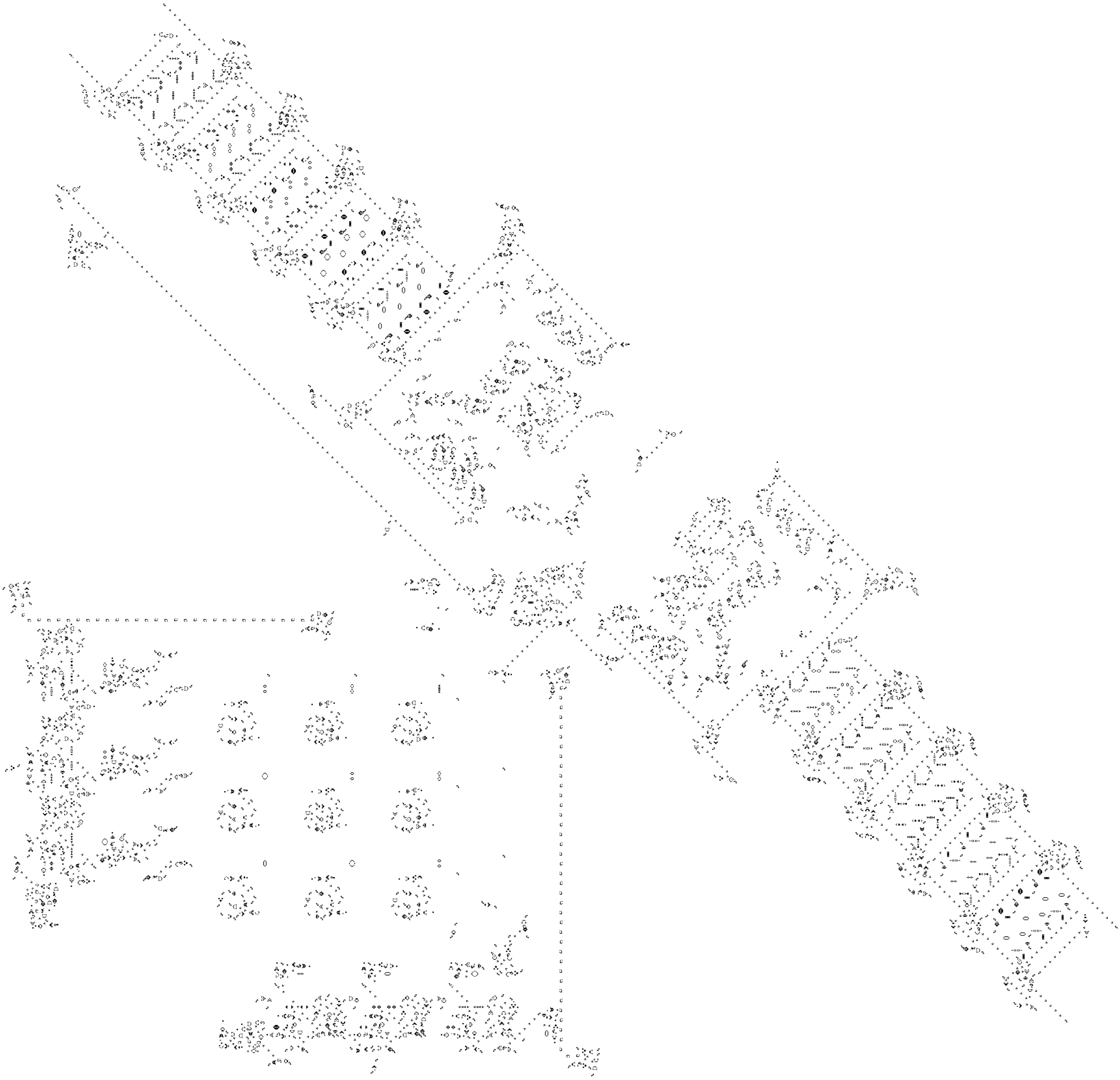


10x10 tatami coverings with exactly 2 monominoes. Is this all of them?



12x12 tatami coverings with exactly 2 monominoes. Is this all of them?

Tutorial 3 – T-Diagrams



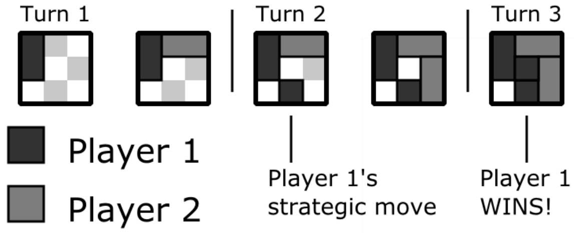
Anecdote: Conway's Game of Life is based on a local rule (situated only around each pixel), but was proven to be able to calculate everything that a computer can calculate, using the "Turing machine" configuration shown above. That is a lot of structure, coming from a simple local rule! For more information Google: Conway's game of life (image credit from www.conwaylife.org)

What is the Purpose of Tutorial 3?

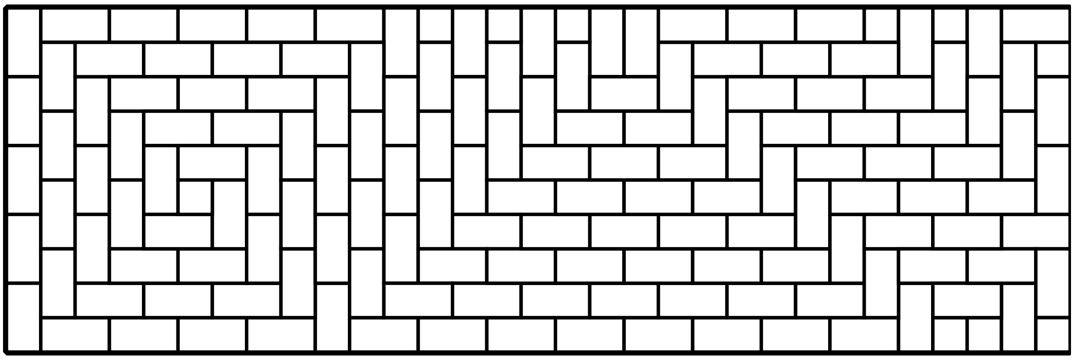
- To abstract the tatami structure into T-diagrams.

Activities

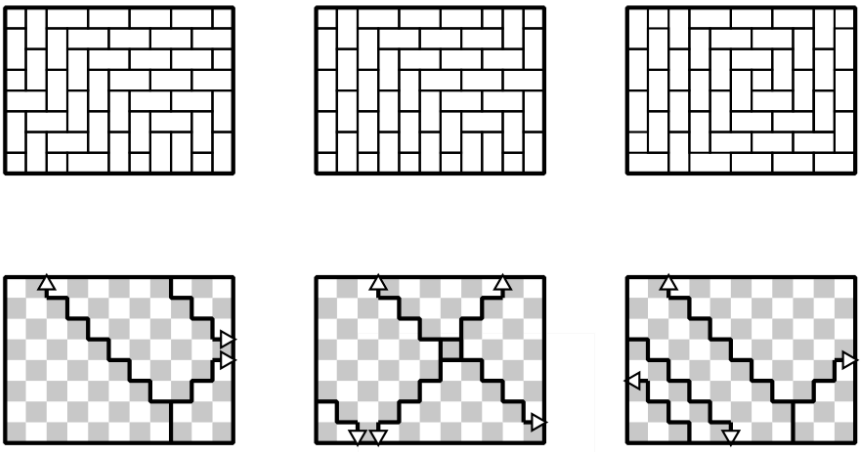
1. **Tatami Oku (warm up)**: The object of this 2 player, head-to-head turn based game is to be the last player who can place a tile on the grid without violating the tatami restriction. With an opponent, alternately place a monomino or a domino on the grid so that **no four tiles meet**. Think strategically so that you prevent your opponent from placing last. Suggested grid size: 3x3 or 3x4.



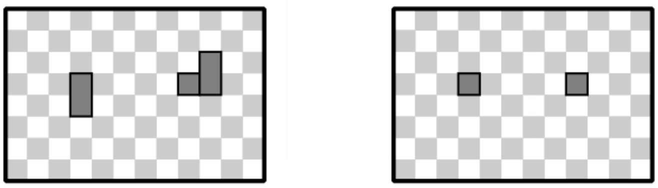
2. **Drawing T-diagrams over coverings.** Draw the T-diagram of this covering (in place).



3. **Matching T-diagrams with their coverings.** You will be given several coverings and T-diagrams, and you need to match them up.



4. **Can these tatami coverings be completed?** You'll be given several partial tatami coverings. Use what you know about T-diagrams to decide whether or not they can be completed. Two examples are shown below. If time allows, you will be asked to make your own partial covering, and see if one of your peers can complete it.

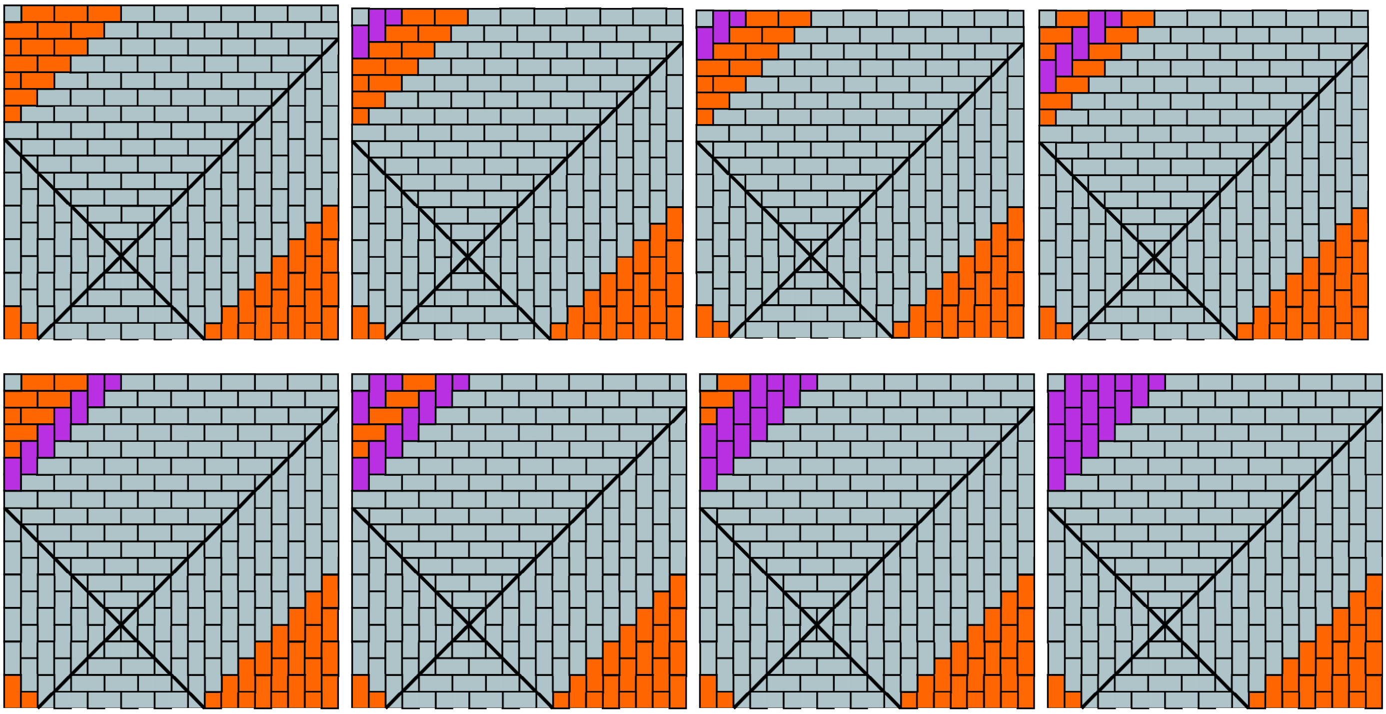


5. **How many binary strings?** Counting binary strings will help us count some tatami coverings later, pay attention! ☺ Here are some examples of binary strings. 0, 1, 01, 000, 10, 1100010, 001010111. The first one is a binary string of length 1, and the last one is a binary string of length 9. How many binary strings of length 5 are there? Of length n? Of length 7 with exactly 3 "1"s?

Homework

1. **Fill in the theorem and proof:** Theorem: You can fit ____ vortices and bidimers in a tatami covering of the square. Proof: ____. Your proof may include drawings.
2. How many tatami domino (only) coverings are there of the chessboard? What about other nxn square grids? What if n is odd?

Tutorial 4 – Count tatami coverings of square grids.

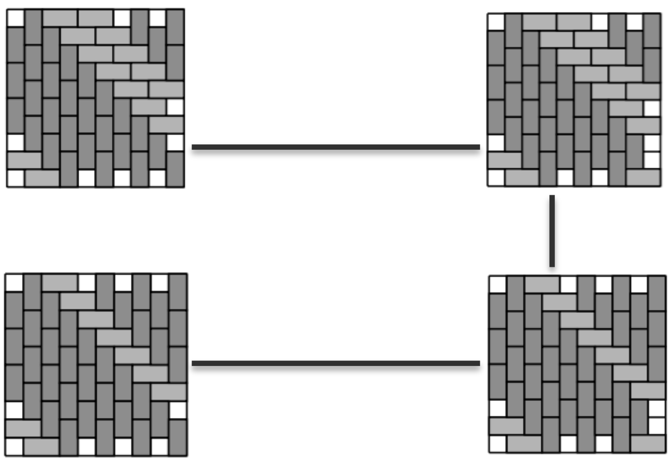


What is the Purpose of Tutorial 4?

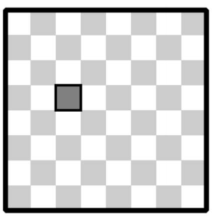
- To assess understanding of and review the Theorem and Proof that T-diagrams represent the whole structure of monomino-domino tatami coverings of grids.
- To apply T-diagrams to count tatami coverings.
- To do research using tables of numerical data.

Activities

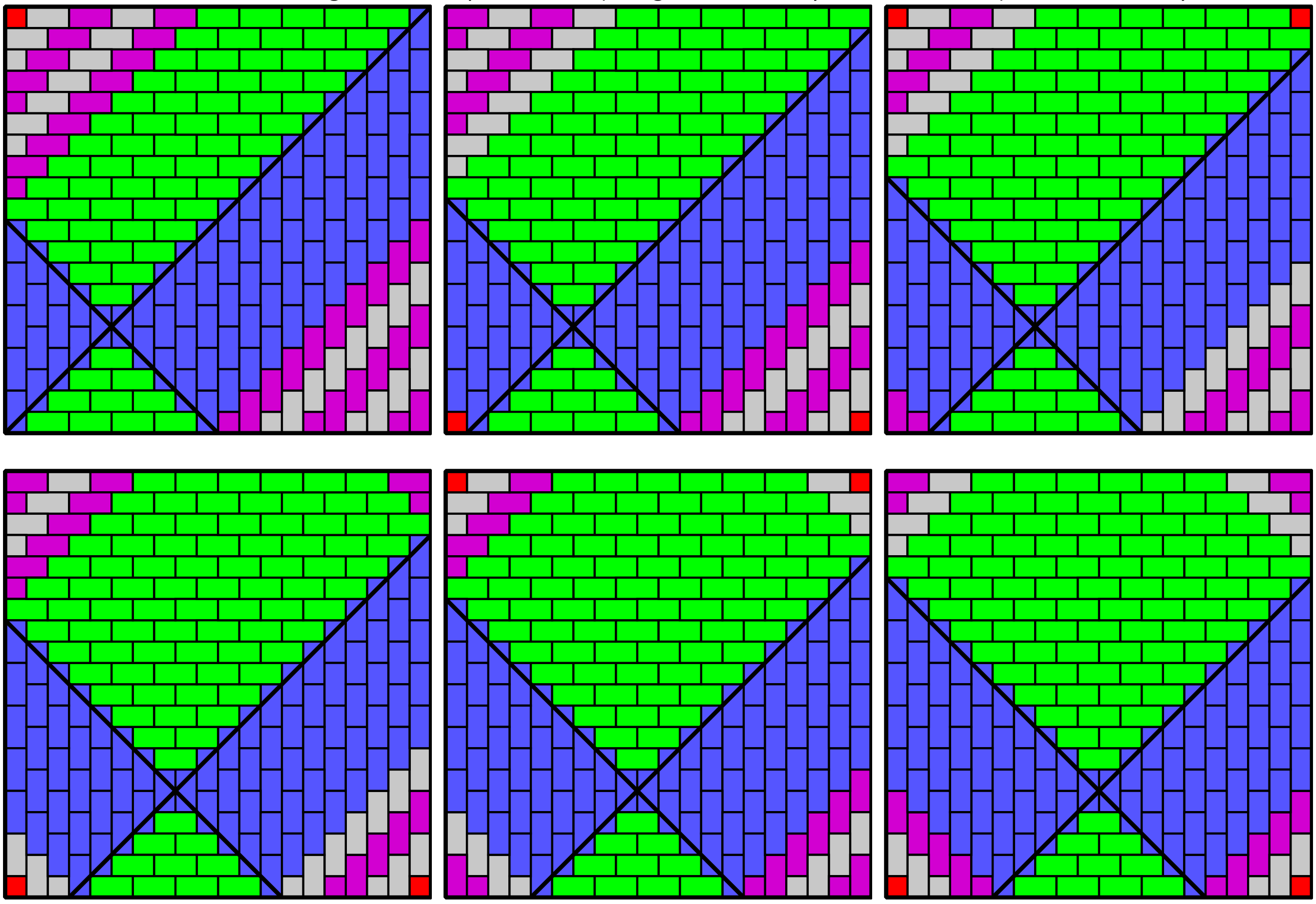
1. A diagonal flip. Connect the dots – draw a line between coverings that differ by exactly one diagonal flip. Examples below.



2. How many tatami coverings are there that complete the one shown below?
    a. Can we use binary strings to count these?



3. **Tatami coverings of the nxn square with exactly m monominoes.** Study these tatami coverings carefully; count the monominoes, the dimensions of the grid, and the positions of the X feature. What do they have in common? Can you infer another covering with these parameters (nxn grid and exactly m monominoes) based on what you see here?



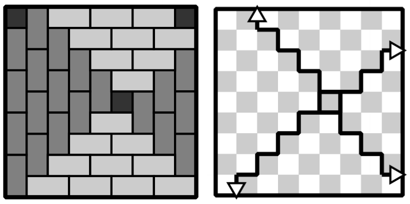
4. Using **OEIS.org**: In Tutorial 2 Homework question 4 we saw how tatami coverings can be related to each other, even when the grids are of different sizes. Specifically, it appeared that the 4x4 grids, 6x6, grids, etc., all have exactly 32 coverings that contain exactly 2 monominoes each. We can also gather important information by looking at the numbers that count tatami coverings, rather than at the coverings themselves. The drawings of n x n coverings shown earlier are the top diagonal of 32s in the table below.
   a. Question: Do you notice any patterns or recognise any sequences in this table? What about the diagonals that seem to stabilise to a constant (32 repeated on the top diagonal, 50 repeated below that, etc)? Why don't the diagonal numbers stabilise to a constant right away?
   b. The Online Encyclopaedia of Integer Sequences (http://oeis.org/) comes in handy when you come across a new sequence (or series) of numbers. Enter a sequence in the OEIS.org search bar and it will try and find it in its database. Try it with the numbers in column 1. Perhaps you've seen these ones before!

$r\backslash c$	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.													
2	1	4												
3	.	9	.											
4	3	18	27	32										
5	.	35	.	52	.									
6	6	64	75	62	60	32								
7	.	112	.	99	.	58	.							
8	10	192	177	152	102	46	50	32						
9	.	323	.	163	.	78	.	50	.					
10	15	534	393	258	184	100	36	36	50	32				
11	.	872	.	343	.	115	.	34	.	50	.			
12	21	1410	829	408	246	182	92	34	18	36	50	32		
13	.	2260	.	632	.	139	.	68	.	18	.	50	.	
14	28	3596	1691	746	414	212	174	92	18	16	18	36	50	32

Table A.3: Number of tatami coverings of the $r \times c$ grid with 2 monominoes, and $r \leqslant c$.

Homework

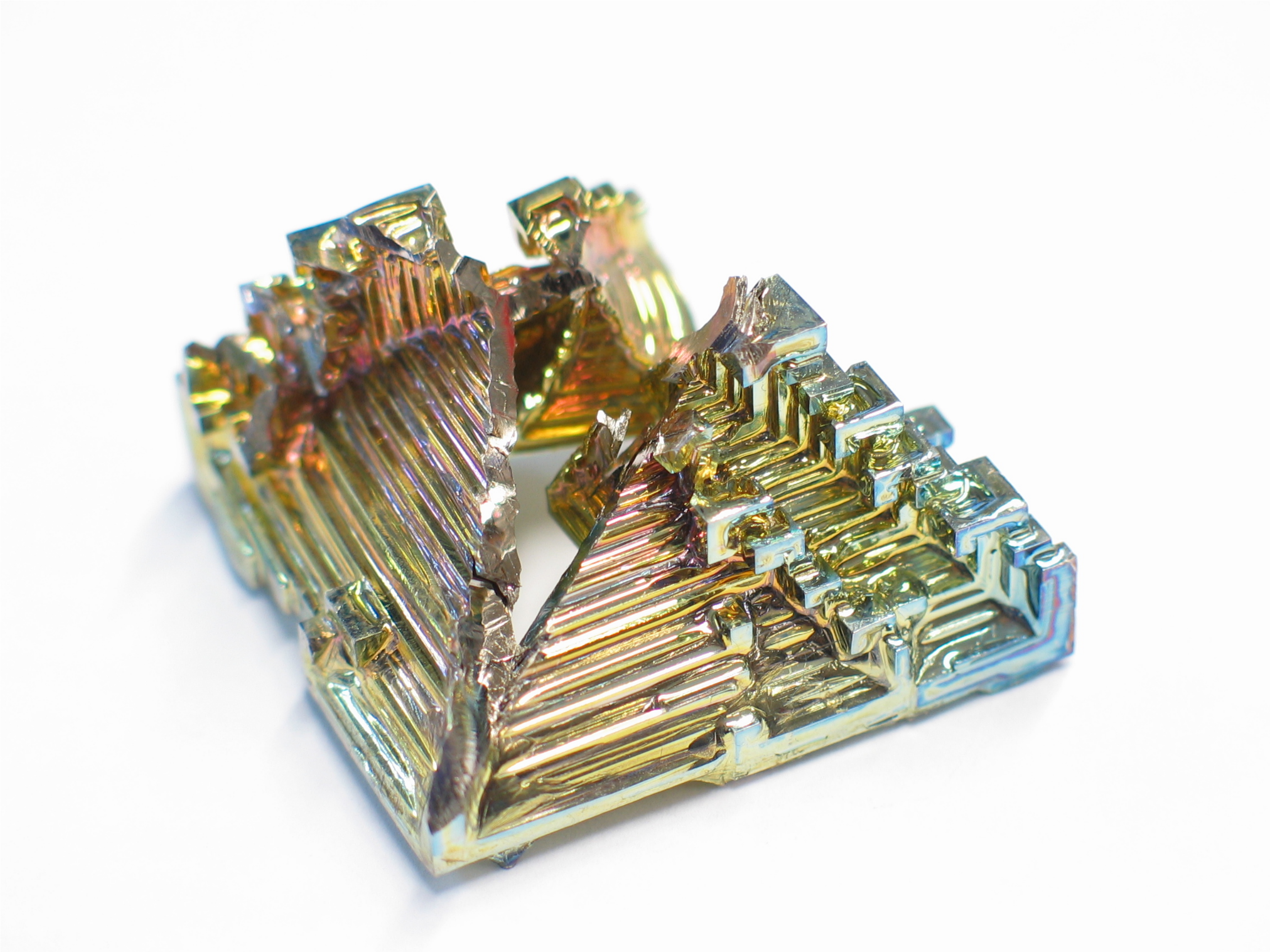
1. **How many tatami coverings of the 9x9 square grid are there with exactly 3 monominoes?** (Illustrate them using T-diagrams).
   Construct your answer in Theorem/proof format. If you are unsure (which is OK!) of the exact number, adjust your theorem statement. For example, you don't need to give the exact number (see below).
   Theorem: There are at least 4 tatami coverings of the 9x9 square grid are there with exactly 3 monominoes.
   Proof: By repeatedly rotating the tatami covering shown below by a quarter turn, we obtain 4 coverings that are distinct from each other. Having found four of the requisite coverings, there must be at least this many. Q.E.D.



2. **Use OIES.org:** The table below shows the number of monomino-domino tatami coverings of the rxc grid (with any number of monominoes), for each r and c up to 14. Do you recognise the sequence of numbers in column 1? Write down what you think it might be called, and then go enter it at http://oeis.org . Put in at least 6 terms; e.g., 1,2,3,5,8,13. The first sequence returned should have the reference number A000045. Copy the name of this sequence into your solution, and 3 facts of your choosing, even if you don't understand what they mean; e.g., "The ratios F(n+1)/F(n) for n>0 are the convergents to the simple continued fraction expansion of the golden section.". Try a few terms of some other rows, columns or diagonals in OEIS and report your findings (in terms of sequence numbers, like A000045).

$r\backslash c$	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1													
2	2	6												
3	3	13	22											
4	5	29	44	66										
5	8	68	90	126	178									
6	13	156	196	238	325	450								
7	21	357	406	490	584	827	1090							
8	34	821	852	922	1165	1404	1914	2562						
9	55	1886	1778	1714	2030	2828	3262	4618	5890					
10	89	4330	3740	3306	3619	4603	6228	7450	10130	13314				
11	144	9945	7822	6246	6080	7890	10226	14979	16734	23730	29698			
12	233	22841	16404	12102	10987	12475	17114	22803	31218	37154	50434	65538		
13	377	52456	34346	22994	19362	20396	25534	38778	50128	74610	81662	115970	143362	
14	610	120472	72004	43682	35477	34708	41034	54826	81298	109569	150114	178050	241410	311298

Tutorial 5 – Local restriction, global structure



What is the Purpose of Tutorial 5?

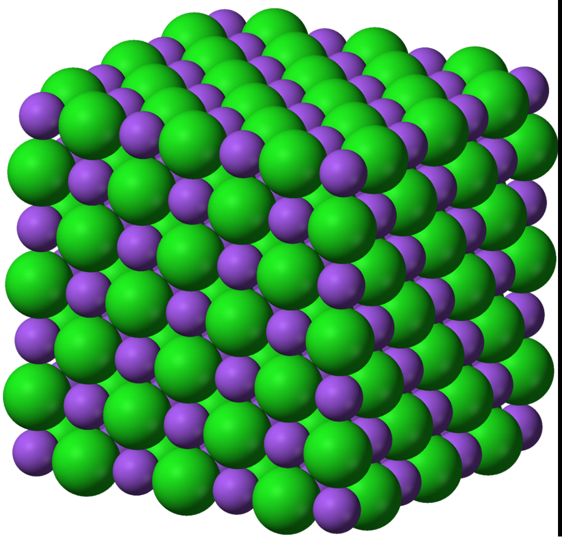
- To see local restrictions and global structure using other tiles.
- To introduce the isometric grid, triangles and lozenges.
- To prepare for and begin the final assignment.

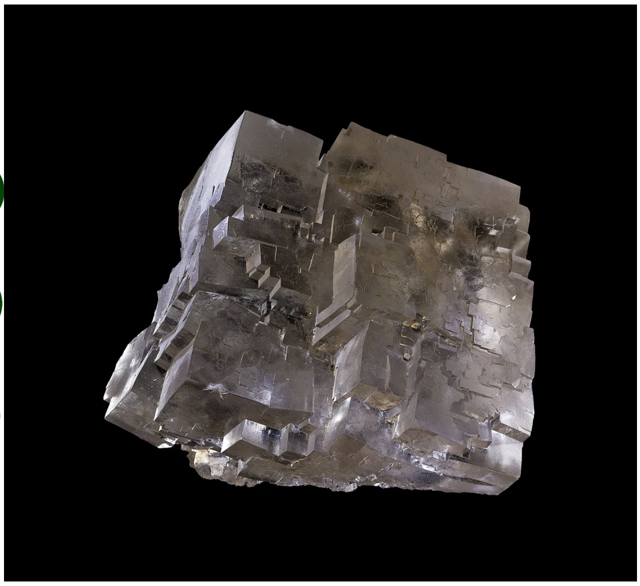
Definition: Other tatami restrictions

The tatami restriction says "no 4 tiles meet", but what if we make a different rule; no 3 tiles meet, for example. Let's call this one the 3-tatami restriction.

Activities

1. **Final assignment:** Read it together. Talk about it. Understand it! The next few activities will help prepare us for the final assignment.
2. **Local rule, global structure:** Local rules that impose a global structure (or order) occur in many places, such as crystal structures (image credit: Wikipedia). The molecules line up *locally* in certain ways, resulting in structures that can be observed *globally*. With a partner, draw a picture of some things that interact according to local rules (involving only the things next to themselves) that have a globally observable structure. Question: Can non-tatami monomino-domino coverings be described by one or more local rules?

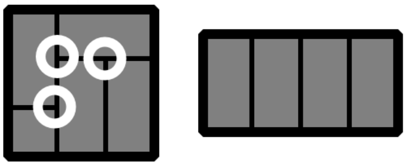




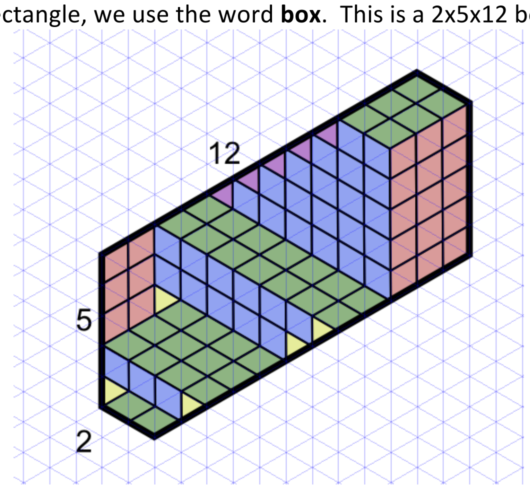
Microscopic structure of a halite crystal. (Purple is sodium ion, green is chlorine ion.) There is cubic symmetry in the atoms' arrangement.

Macroscopic (~16cm) halite crystal. The right-angles between crystal faces are due to the cubic symmetry of the atoms' arrangement.

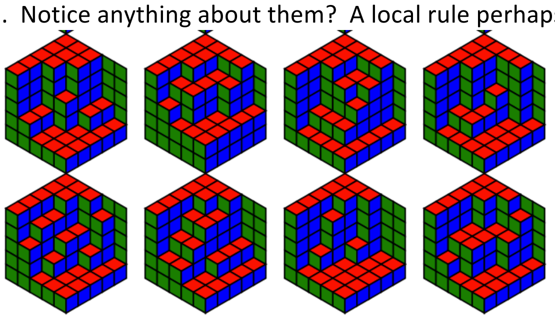
3. **A 3-tatami restriction?** What sort of global structure does the 3-tatami restriction impose? The (4-)tatami covering below left violates the 3-tatami restriction in 3 places. The covering below right is a 3-tatami covering of the 2x4 grid.



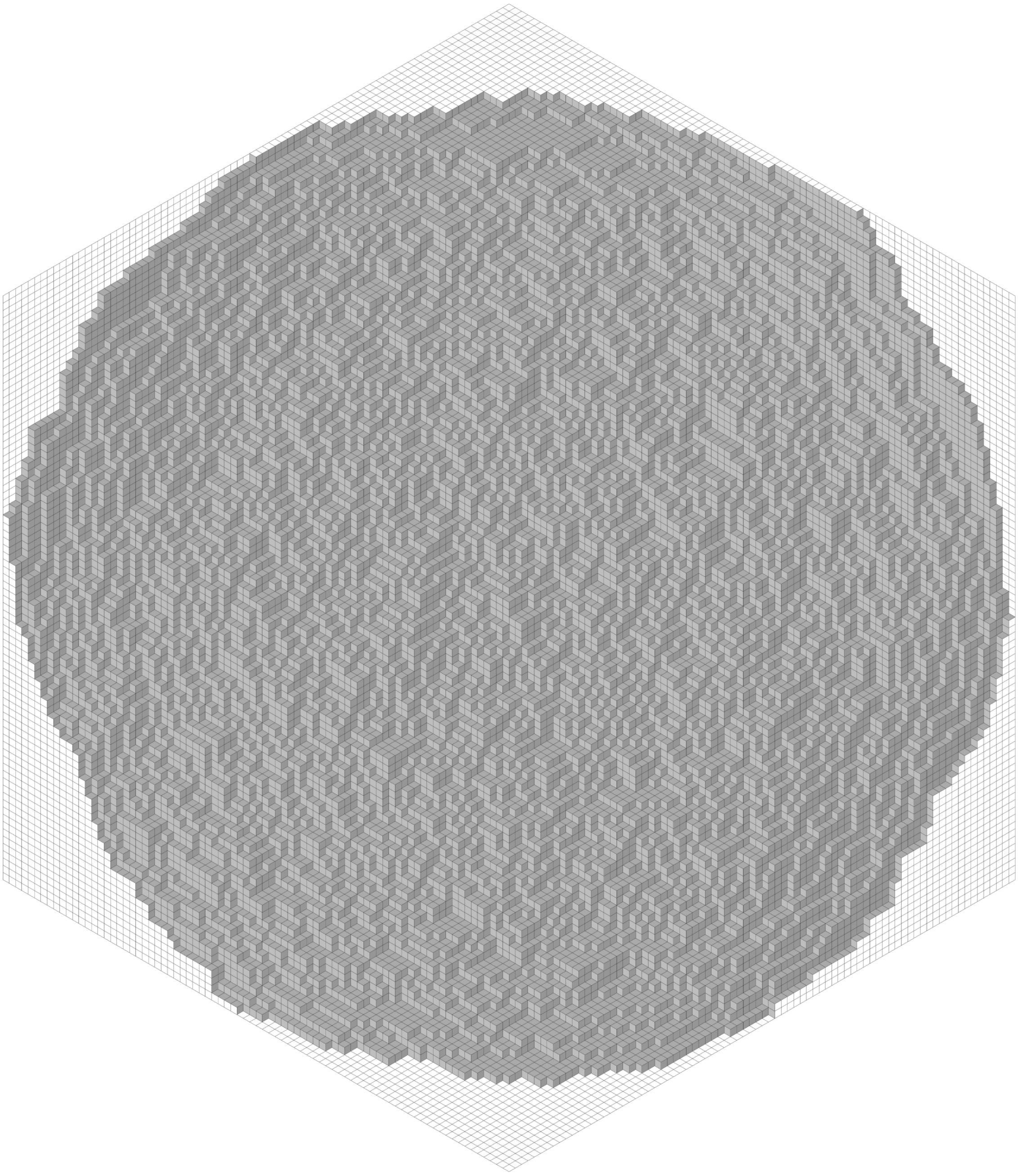
4. **Lozenge coverings:** So far we've looked only at monomino-domino coverings of the square grid. Now we will look at coverings of the isometric grid (from equilateral triangles) by triangles and lozenges (2 triangles together). Instead of rectangle, we use the word **box**. This is a 2x5x12 box.



Amazingly, all lozenge (only) coverings of boxes **look like a stack of cubes shoved into a corner**. Here are some that I created using a computer program. Notice anything about them? A local rule perhaps?



Even more amazingly, is that **most lozenge coverings of the regular hexagon form what mathematicians call an Arctic circle,** like the one shown below. This was proven in the Arctic Circle Theorem (image credit to Wikipedia).



5. **Lozenge tatami practice and start the final assignment.**

Final Assignment: The tatami restriction at large

We have learnt about a tatami restriction that applies to monomino-domino coverings of rectangular grids. The tatami restriction is combinatorially interesting because it is a very simple, local rule -- not four tiles may meet at any point -- which imposes a rich global structure.

This time we will invent new tatami restrictions for coverings of the isometric grid with triangles and lozenges. There are several different choices of tatami restriction here; we may disallow 2,3,4,5, or 6 tiles to meet (at grid intersections). Let us name these tatami restrictions as follows:

2-tatami	No 2 tiles may meet at any point.
3-tatami	No 3 tiles may meet at any point.
4-tatami	No 4 tiles may meet at any point.
5-tatami	No 5 tiles may meet at any point.
6-tatami	No 6 tiles may meet at any point.

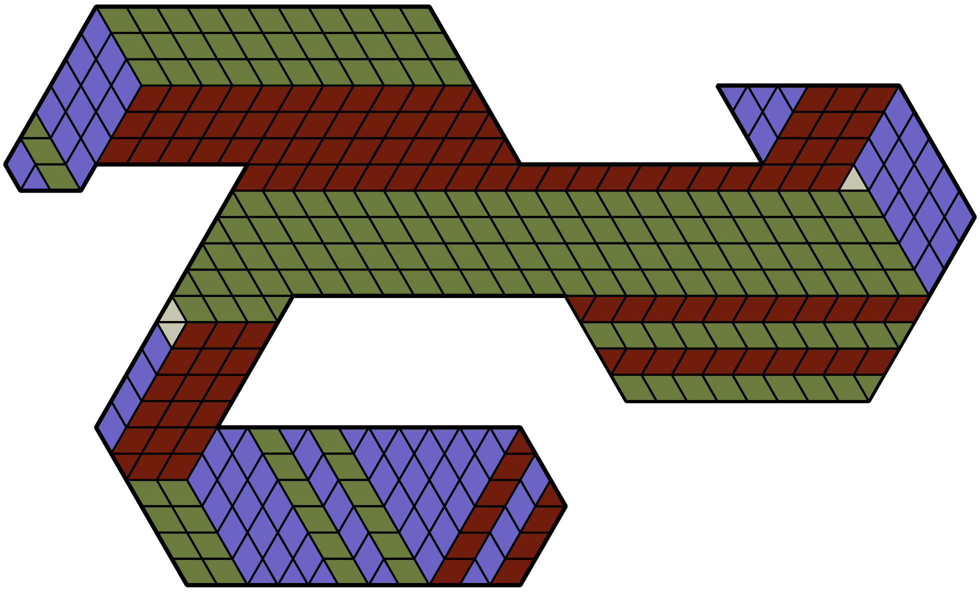
Some of these are too restrictive to be combinatorially interesting; e.g., a box on the isometric grid cannot be covered and satisfy the 2-tatami restriction unless it consists of a single grid-triangle or no grid-triangles (a vacuous box!). Thus, we would argue that the 2-tatami restriction is combinatorially uninteresting.

Part 1: For each of the 3,4,5, and 6-tatami restrictions argue why or why not they may have a combinatorially interesting structure. You are encouraged to use drawings, and write in full sentences to support your ideas. Questions to think about: Is the rule so restrictive that very few coverings are possible? Why is this not combinatorially interesting? Is the rule rather unrestrictive, so that almost every covering is possible? Note that 5-tatami and 6-tatami (isometric) coverings appear in the activities for Tutorial 5.

Part 2: For the 5-tatami restriction draw and describe as much of the structure as you can (Hint: Argue in Part 1 that this one *is* combinatorially interesting. Use a Theorem/proof style that we used for the original tatami structure in Tutorials 2-3. Use the isometric grid paper provided in the Appendix in order to draw configurations of lozenge and triangle tiles that make up the structure, as well as (a new version of) "T-diagrams", where appropriate.

Write maximum 2000 words, plus figures. Any figures should be labelled, Figure 1, Figure 2, ..., and you should write "(see Figure x)" in the text to refer to them. They do not need to appear directly in a typed document, but all figures must be referred to in the text.

Tutorial 6 – Isometric grids together



What is the Purpose of Tutorial 6?

- To reflect on the final assignment.
- To walk through the lozenge tatami proof together.
- To identify and correct any misunderstandings about the material.
- To think about future research directions for tatami coverings.

Activities

1. **Reflection on the final assignment and course:** What did you find most difficult? Easiest? What did you enjoy? What questions would you add to the final assignment?
    a. **1 minute summary:** Summarise Part 1 of the final assignment in 1 minute, using tile coverings that support your reasoning.
    b. **Exit tickets:** Write down one concept or fact that you are confident about, and one that you are unsure of.
2. **Share your discoveries:** You will have the opportunity to share your discoveries in the final assignment with your peers. We will use this activity to collaborate on a complete proof of the triangle-lozenge 5-tatami covering structure.
3. **All questions answered:** I will answer your questions about tatami coverings and related things.
4. **Future research:** A "Future research" section is common in academic reports and articles. It is a description of what the researcher thinks should be researched next, by considering what sorts of answers are achievable in the near future, or what open questions most important. For example, one item of future research could be "Design a computer program to create drawings of all tatami coverings of the 6x6 square grid". **What do you think should be future research on tatami coverings?**

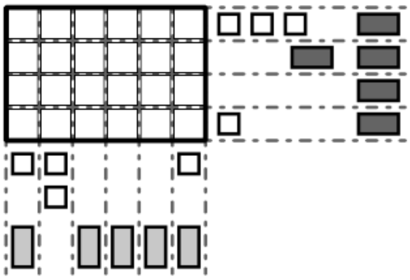
Appendix

Tomoku

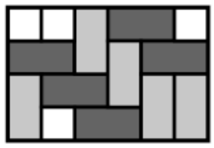
Here are the full instructions for tomoku and several extra puzzles to complete.

Tomoku!

Mats are shown in the Tomoku puzzle beside their containing column or below their containing row. Notice that **a □ appears in both a row *and* column**. You must check both places before placing a □. The order of the mats within each row and column does not matter.

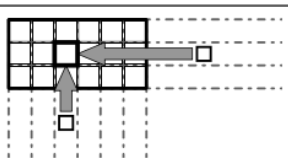


Tomoku Puzzle.

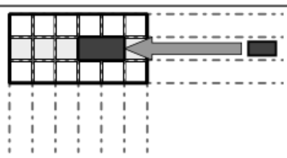


Lucky Solution.

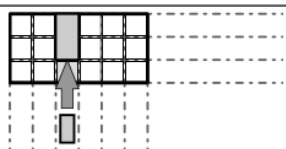
The object of the Tomoku puzzle game is to find a lucky layout using the mats shown in the rows and columns of the puzzle.



A □ appears in its **row and column.**

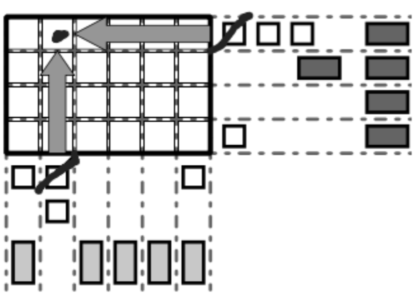


A ▬ stays in its row.

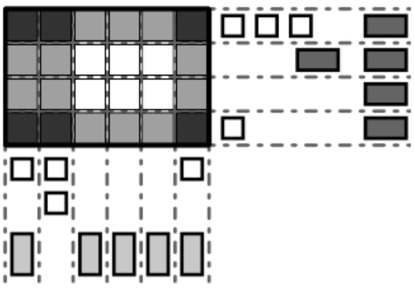


A ▯ stays in its column.

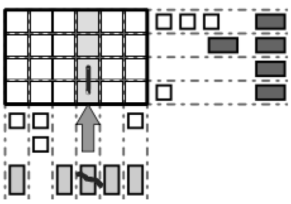
To fill in a puzzle, use a short line to represent a ▬ or ▯ and a dot to represent a □. Cross off mats you have used in the arrangement. To place a □ you must use one from both a **row and a column**.



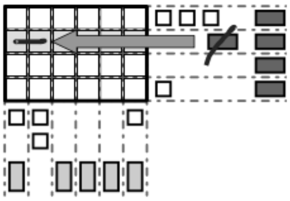
Adding a □ uses one in both a **row and a column**.



A □ may only be placed on the dark gray squares.

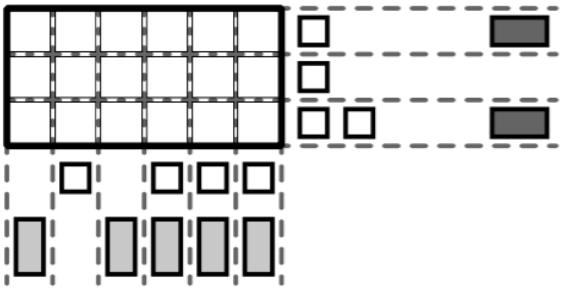


Adding a vertical mat.

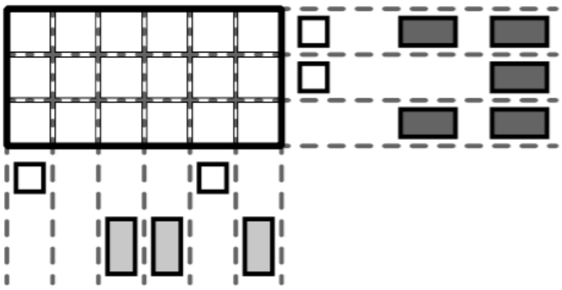


Adding a horizontal mat.

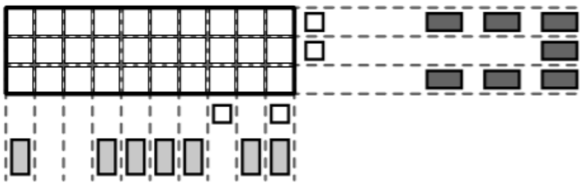
Completed puzzle.



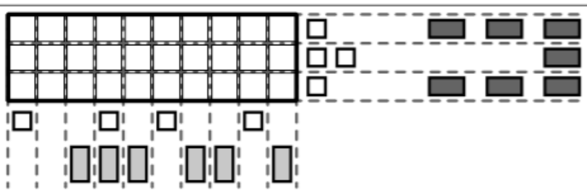
Puzzle 7



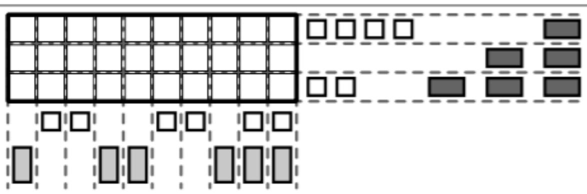
Puzzle 8



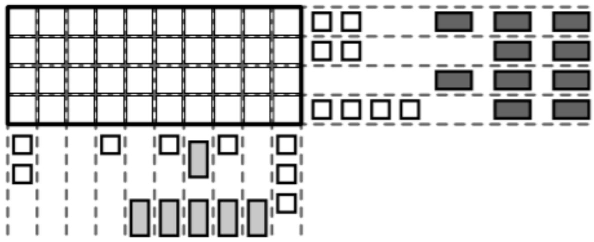
Puzzle 11



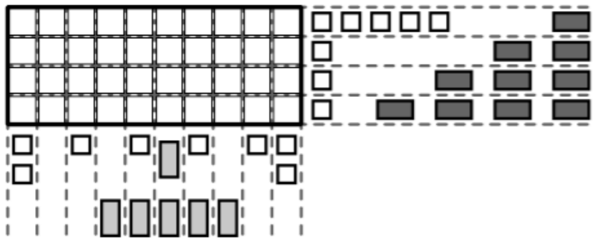
Puzzle 12



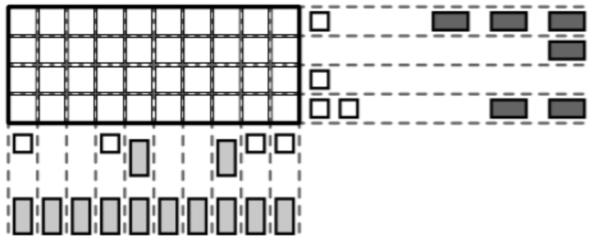
Puzzle 13



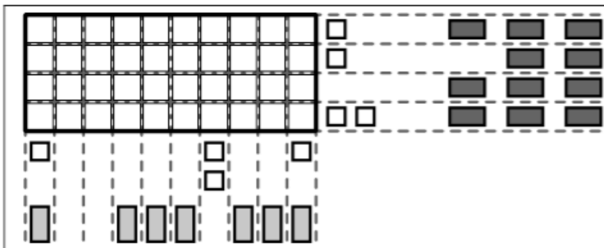
Puzzle 28



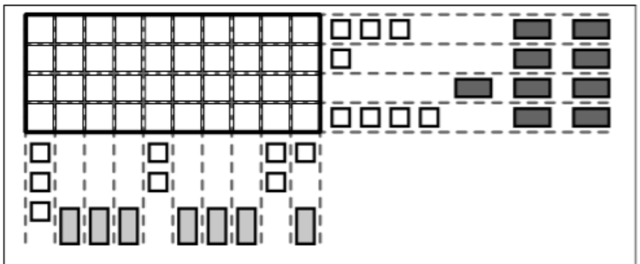
Puzzle 29



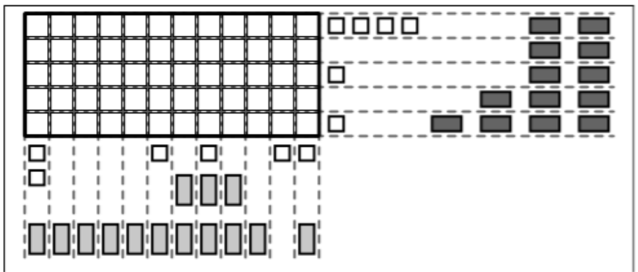
Puzzle 24



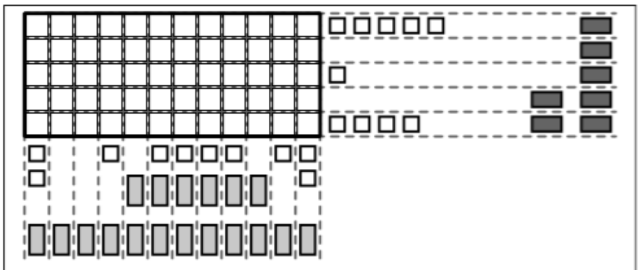
Puzzle 25



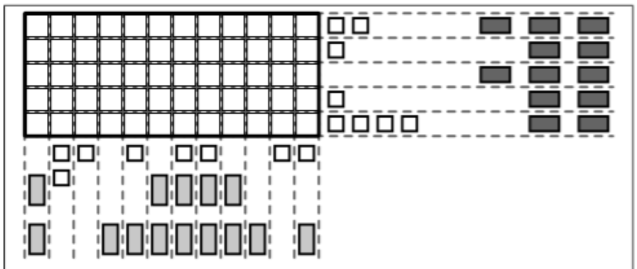
Puzzle 30



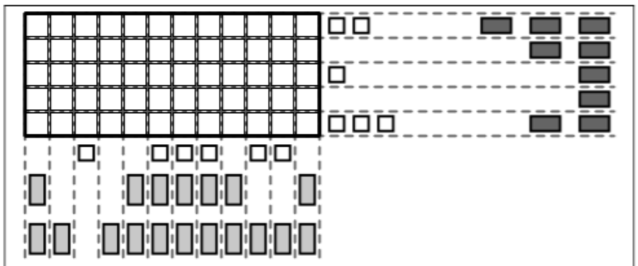
Puzzle 42



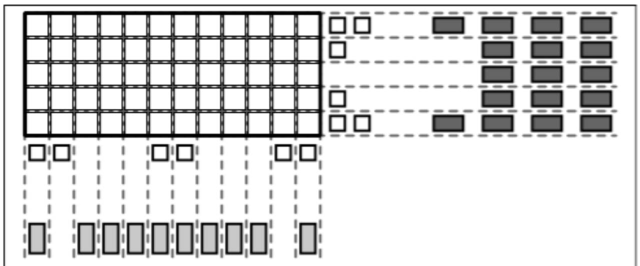
Puzzle 31



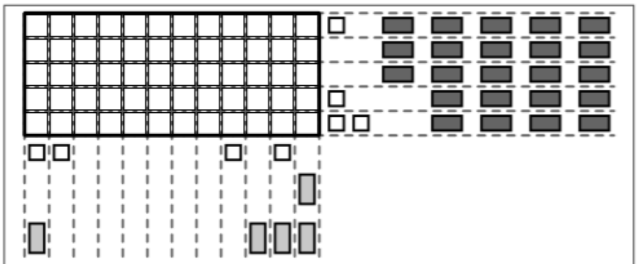
Puzzle 43



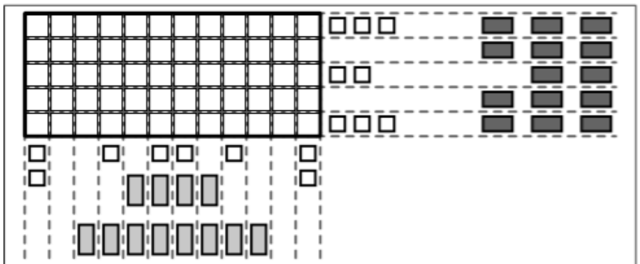
Puzzle 36



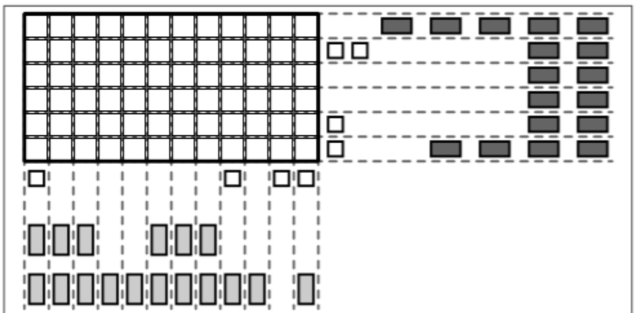
Puzzle 38



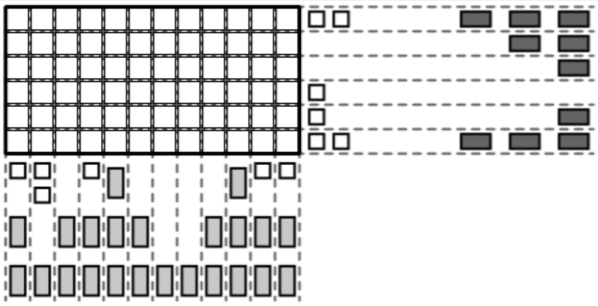
Puzzle 37



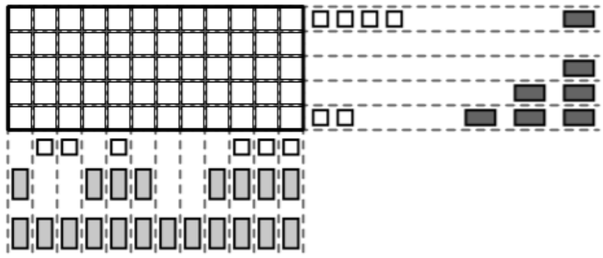
Puzzle 39



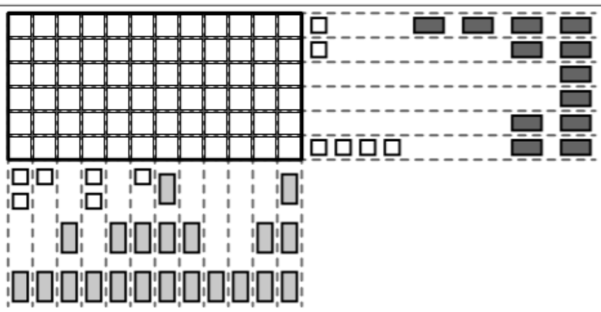
Puzzle 53



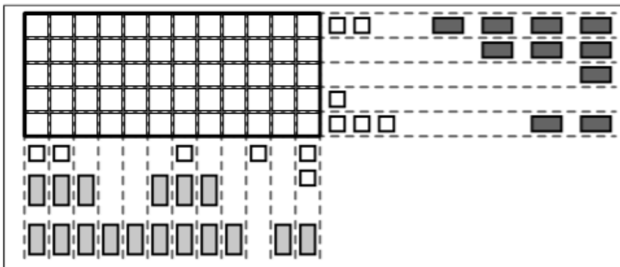
Puzzle 56



Puzzle 46



Puzzle 57



Puzzle 47

Square grid paper

Isometric grid paper

The Scholars Programme